A proposal to measure the position of the DECam camera with respect to the Blanco primary mirror

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ABSTRACT

This note is intended to stimulate the discussion about using lasers mounted in the DECam camera as a way of measuring the position of the camera relative to the primary mirror. At this stage there are many question that need to be resolved before being able to know what the best option is, but we feel that the process might be shorten if some of our DES collaborators have the opportunity to comment on the proposal at this early stage.

1. INTRODUCTION

The current plan to measure the relative position between the CCD focal plane array and the primary mirror is based on using eight focusing and alignment CCDs placed around the periphery of the physics CCDs.[1] After a 100 second exposure the focusing and alignment CCDs can provide lateral alignment information to an rms accuracy of 100 μm[2,3] (with tip, tilt and focusing errors unknown to us).

The current specifications for the hexapod positioning system require errors of ± 25 μm in translation, ± 5 μm in focusing and ± 1 arcsec in tip and tilt[4,5] (which corresponds to rms values of 14 μm, 3 μm and 0.6 arcsec respectively). With these set of specifications the hexapod will be able to position the camera with an accuracy which appears to be beyond what can be measured with the focusing and alignment CCDs.

The object of this note is to begin to explore the possibility of using lasers attached to the camera and/or the primary mirror to try to determine the camera position with respect to the primary mirror. We will explore three different arrangements of lasers and CCDs, one of them in combination with BCAMs[7,8] , and a fourth arrangement where only BCAMs are used. In general we will use the name CCD to refer to a device capable of measuring the position of a well collimated beam of light with high accuracy.

In our preferred arrangement the lasers will be attached to the outside of the camera vessel near the focal plane and they will point towards the primary mirror. Three of the laser beams will be reflected from the primary mirror and the other three will be reflected from three small flat mirrors solidly attached to the outside edge of the primary mirror. CCDs (or similar devices) near each laser will accurately measure each reflected beam. We believe that if this scheme can be made to work it has the great advantage of directly measuring the position of the camera with respect to the primary mirror and of being completely independent of the primary mirror and/or camera support structures.

If we have to use visible light lasers then the system will be used while the shutters are closed and the focal plane CCDs are being red out. If we find out that we can use 1.5 μm infrared lasers then the system could be active at all times.[6] It is likely that the resolution will be ultimately determined by the laser beam dispersion in air. In Section 2 we will give a brief conceptual outline of several arrangements of lasers, CCDs and BCAMs that could be used to measure the position of the camera with respect to the primary mirror. In Section 3 we will describe in detail the different arrangements and we will calculate the relation between the errors in the laser beam position measurement and the errors in the camera position determination. In Section 4 we will comment on the main issues that need to be studied in order to understand what the resolution of these systems will be.

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2. CONCEPTUAL OPTIONS

In this section we will describe several conceptual lasers, CCDs and BCAM arrangements as a way to explore the one that could give us the best resolution and construction simplicity. For clarity it will be useful to start the discussion thinking only about measuring two degrees of freedom. As the two degrees of freedom we will choose the camera's translation along the x-axis and the rotation around the y-axis. Both of these transformations move the laser beam along x. We will assume a parabolic primary mirror with a focal length $F = 10$ meters. The coordinate system will be define with the z-axis along the parabola's axis, the origin of the coordinate system will be at the center of the parabola, the focal plane will be at $z = 10$ meters and rotations and translations will be performed at the focal plane. In this coordinate system the parabola equation is $z = (x^2 + y^2)/4F$.

2.1 Option 1

At the focal plane and $x \approx 0.7$ meters we will have a laser paired with a CCD. The laser is pointing down and the angle of the laser beam will be such that after the beam hits the parabolic mirror (at $\approx$1.4 meters) it will reflect back to the CCD that is right next to the laser. The CCD will measure the beam position S_1.

If we translate the camera by Δx the laser beam moves parallel to itself and due to the focusing properties of the mirror the reflected beam will hit the focal plane at exactly the same position as it did before the translation. Since the CCD is also attached to the camera the beam spot on the CCD will move by an amount $\Delta S_1 = -\Delta x$.

If we rotate the camera around the y-axis by an amount $\Delta\theta$ the laser will rotate towards the vertical axis and the beam will move towards smaller values of x. The change in x is easy to calculate because for small displacements and small angles the parabolic mirror acts as a thin lens with focal length F. The beam going through the center of the lens is not deflected and therefore the position at the focal plane will move by $F\Delta\theta$. All parallel rays rotate by the same angle and will end up at the same point. Therefore the laser beam at the CCD will move by an amount $\Delta S_1 = -F\Delta\theta$. It we use angular units of arcseconds (1 arcsec = 4.85 $\times 10^{-6}$ μrad) and express translations in microns we will have $\Delta S_1 = -48.5\,\Delta\theta$. The result of combining rotations and translation is

$$\Delta S_1 = -\Delta x - 48.5\,\Delta\theta \tag{1}$$

Now we need more equations if we want to solve for $(\Delta x, \Delta\theta)$ so we may think about installing more lasers close to the focal plane and reflecting them from the primary mirror. Unfortunately this doesn't work because they are all degenerate. To illustrate this we can think about installing a second pair laser/CCD at $x \approx -0.7$ meters. For a translation Δx the reflected beam will again stay in place while the CCD moves with the laser, so as before we will have $\Delta S_2 = -\Delta x$. If we rotate the camera by $\Delta\theta$ the beam for the laser at negative x will move away from the parabola's axis and the reflection will become more negative giving $\Delta S_2 = -F\Delta\theta$ as before. Therefore the combination of rotations and translation will gives an equation identical to Eq. 1, clearly showing that adding more laser/CCD pairs reflecting out of the primary mirror will not help. The degeneracy breaks slightly if the laser/CCD pairs are positioned at different z heights, like for example one pair is at the beginning of the camera and the other at the end of it. In this case the degeneracy breaks weakly, so it is possible to determine translation and rotations but the resolution is very bad.

One way to completely break the degeneracy is to reflect from a flat mirror. In this case a translation will have no effect, while a rotation will produce a beam motion at the CCD of $\Delta S_2 = -2F\Delta\theta$. Therefore if laser 1 is reflected from the primary mirror and laser 2 from a flat mirror rigidly attached to the primary mirror we will have

$$\begin{pmatrix} \Delta S_1 \\ \Delta S_2 \end{pmatrix} = \begin{pmatrix} -1.00 & -48.5 \\ 0.00 & -97.0 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta\theta \end{pmatrix} \tag{2}$$

which readily inverts to

$$\begin{pmatrix} \Delta x \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} -1.00 & 0.50 \\ 0.00 & -1/97 \end{pmatrix} \begin{pmatrix} \Delta S_1 \\ \Delta S_2 \end{pmatrix} \tag{3}$$

If σ_{S1} and σ_{S2} are the rms resolution of the CCDs, and $\sigma_{S1} = \sigma_{S2} = \sigma_S$ then the resolution for determining the camera's translation and rotation is $\sigma_x = \sqrt{1^2 + 0.5^2}\ \sigma_S = 1.12\ \sigma_S$, and $\sigma_\theta = 0.01\ \sigma_S$. It is not very relevant where the flat mirror is positioned as long as it is solidly attached to the primary mirror. Perhaps a reasonable place to install it would be at the edge of the primary mirror.

2.2 Option 2

It is not difficult to achieve micron level resolution with a CCD, so most likely the final accuracy in determining the laser beam position will be given by other factors like dispersion of light in air. So the first modification to the example given above would be to replace the flat mirror by a CCD and measure one of the laser beams directly (instead of reflecting it from a flat mirror). The advantage of doing this is that the path in air is shorter by a factor of two. In this case $\Delta S_2 = \Delta x$ for translations and $\Delta S_2 = -F\Delta\theta$ for rotations. Here we have assumed that the CCD is at the parabola's origin. If the CCD is mounted at the edge of the primary mirror the relation $\Delta S_2 = \Delta x - F\Delta\theta$ will be slightly modified because the CCD will be at a different hight and it will be a bit angled. Since we are mostly investigating concepts we will ignore these minor changes. Then we have

$$\begin{pmatrix} \Delta S_1 \\ \Delta S_2 \end{pmatrix} = \begin{pmatrix} -1.00 & -48.5 \\ 1.00 & -48.5 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta\theta \end{pmatrix} \tag{4}$$

which inverts to

$$\begin{pmatrix} \Delta x \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} -0.50 & 0.50 \\ -1/97 & -1/97 \end{pmatrix} \begin{pmatrix} \Delta S_1 \\ \Delta S_2 \end{pmatrix} \tag{5}$$

This gives a resolution $\sigma_x = 0.7\ \sigma_S$, and $\sigma_\theta = 0.015\ \sigma_S$. We can see that in this case the position resolution is better than in Option 1, and the angular resolution is slightly worse (but still much better that what is needed). The advantage of Option 1 is that the lasers and CCDs are all in one place and that it may be easier to attach a flat mirror at the edge of the primary than a CCD with the corresponding electronics. Also if the mirror can rotate relative to the camera then we will have to add another laser pointing towards the focal plane with the corresponding CCD. The reason is that if the camera is fixed a rotation of the mirror will only rotate the CCD attached to it very slightly and we will see no effect.

2.3 Option 3

In the previous option we replaced the flat mirror by a CCD and that way we measured the laser beam position directly. If one adds a lens of focal length f at a distance f in front of the CCD then one measures the beam angle instead of the position. This is what the "Brandeis CCD Angle Monitor[7,8]" (or BCAM) does. Each BCAM has two 875 nm LEDs, a light diffuser, a lens and a CCD. When pointed at each other they can measure their relative transverse position by looking at the beam spot displacement on the CCD. Since the beam going through the center of a lens doesn't deflect, the distance D between two BCAMs facing each other, their transverse displacement Δx and the beam displacement ΔS at the CCD are related by $\Delta x/D = -\Delta S/f$. The BCAM light beams have a very large divergence and the part of the beam that is seen by the CCD is determined by a small aperture in front of the lens.

If we replace laser 2 by a highly divergent light beam and the flat mirror by a BCAM then when the camera is translated by a distance Δx the beam at the CCD will be displaced by $\Delta S_2 = -(f/F)\Delta x$. On the other hand if the camera is rotated by $\Delta\theta$ then all that will happen is that BCAM will just pick up a different part of the laser beam and the spot at the CCD will not move, that is $\Delta S_2 = 0$. A typical value of f for BCAMs is 0.15 meters, so this combination of laser 1 and BCAM will give

$$\begin{pmatrix} \Delta S_1 \\ \Delta S_2 \end{pmatrix} = \begin{pmatrix} -1.000 & -48.5 \\ -0.015 & 0.0 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta\theta \end{pmatrix} \tag{6}$$

which inverts to

$$\begin{pmatrix} \Delta x \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} 0.000 & -66.70 \\ -0.021 & 1.37 \end{pmatrix} \begin{pmatrix} \Delta S_1 \\ \Delta S_2 \end{pmatrix} \tag{7}$$

A BCAM resolution of $\sigma_{S2} = 0.5$ μm is typical, so event if ΔS_1 is very course we will get $\sigma_x = 33$ μm and $\sigma_\theta = 0.7$ arcseconds. If three pairs of BCAMs are used then the transverse resolution will likely reach the 15 μm level in x, which matches the hexapod specification. This option will work, the only thing that needs to be investigated further is that if the resolution in z will match the hexapod specification.

2.4 Option 4

David Gerdes has looked into the possibility of using BCAMs to measure the position of the camera.[9] So the obvious question now is, can we use only BCAMs to measure where the camera is at? In order to measure position and angles we will have to add another BCAM at a different hight. As in Option 3 a BCAM attached to the edge of the primary mirror looking at a BCAM positioned at the focal plane will be sensitive to translation according to $\Delta S_1 = -(f/F)\Delta x$. A BCAM situated at a distance L from the focal plane will see both rotation and translations as $\Delta_T = L\,\Delta\theta + \Delta x$. And the beam at the BCAM CCD will move by $\Delta S_2 = -[f/(F-L)](\Delta x + L\,\Delta\theta)$. We will assume L= -1 meter, and as before we will use f=0.15 meters. Therefore in units of microns and arcseconds we get

$$\begin{pmatrix} \Delta S_1 \\ \Delta S_2 \end{pmatrix} = \begin{pmatrix} -0.0150 & 0.000 \\ -0.0167 & -0.081 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta\theta \end{pmatrix} \tag{8}$$

which inverts to

$$\begin{pmatrix} \Delta x \\ \Delta\theta \end{pmatrix} = \begin{pmatrix} -66.67 & 0.00 \\ 13.75 & -12.34 \end{pmatrix} \begin{pmatrix} \Delta S_1 \\ \Delta S_2 \end{pmatrix} \tag{9}$$

A typical BCAM resolution is $\sigma_S = 0.5$ μm giving a resolution of $\sigma_x = 34$ μm and $\sigma_\theta = 6$ arcsec for translations and rotations respectively. The reason the angular resolution is so much worse then in previous cases is due to the fact that L can not be very big.

2.5 Final remarks

Some final remarks are in order here. The camera position is transmitted to both the laser beam position and its direction. To measure the position all we need to do is to intercept the laser with a CCD. To measure the angle we need a lens to convert angles into positions and then put a CCD.

The lens to transform direction into position can be the BCAM lens or the primary mirror. Each one has its own advantages and disadvantages. In the case of BCAM there are little demands on the laser but the resolution is sacrificed because the lens is very close to the CCD. When using the primary mirror as "the lens" we have an optimal configuration because "the lens" is half way between the laser and the CCD, on the other hand the demands on the laser are larger because to keep the beam spot small we need laser beams with small divergence. If the limiting factor is the CCD resolution then using the primary mirror as the lens will give the smallest camera positioning errors. If the limiting factors are things like dispersion in air or laser beam stability, then we need further investigation to determine the best configuration.

3. REALISTIC CONFIGURATIONS

In this section we will extend the conceptual examples of Section 2 to more realistic cases. The number of lasers will be increased to six, and we will be measuring the six degrees of freedom of the relative position between the camera and the primary mirror. As before we will consider four options:

- Option 1.

  We will use six lasers attached to the camera near the focal plane. Three of them will point to the primary mirror and the other three to flat mirrors attached to the outside of the primary mirror. The reflexions will be measured by devices installed very close to each of the lasers, we will refer to these devices as CCDs. For simplicity in the calculations, in this section we will assume that the CCDs and the lasers are in the same position. Displacing the CCD slightly (in comparison to the distance between the camera and the primary mirror) will not change the conclusions of this section. The position and direction of the lasers are given in

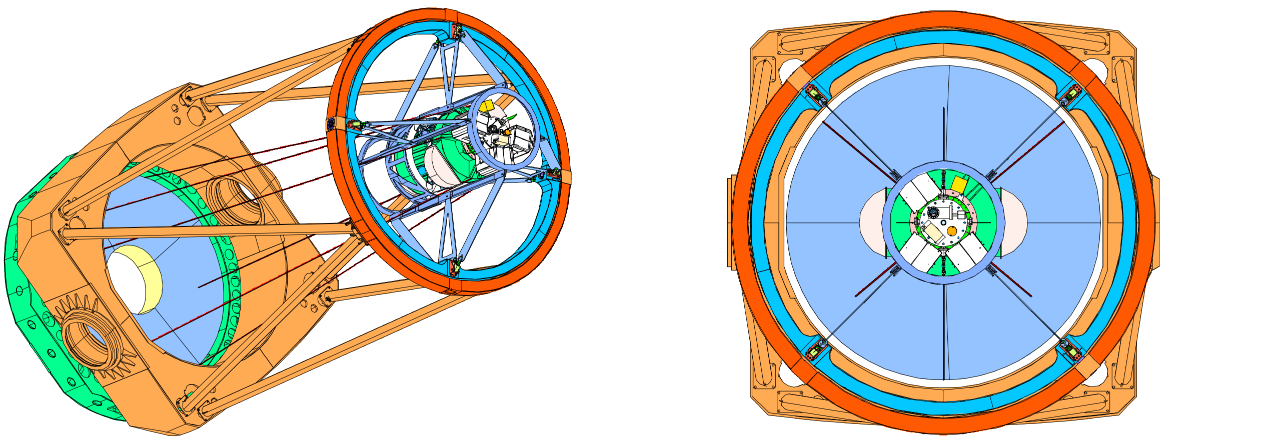

Figure 1. Blanco telescope figures showing the alignment laser beams in red. The figure on the left shows the laser beams going from the camera towards the primary mirror. The figure on the right is a top view of the telescope, we can easily see that three of the lasers hit the primary mirror and the other three hit the outside edge of the mirror.

Table 1. Position and direction of lasers in Options 1 and 2. In Option 3 lasers 4 to 6 are replaced by BCAMs in the same position and pointing in the same direction.

	r	ϕ	α	D	x	y	z	n_x	n_y	n_z
laser 1	735	40°	4.181411°	9973	563.04	472.45	10000	0.055856	0.046869	-0.997338
laser 2	735	140°	4.181411°	9973	-563.04	472.45	10000	-0.055856	0.046869	-0.997338
laser 3	735	270°	4.181411°	9973	0.00	-735.00	10000	0.000000	-0.072915	-0.997338
laser 4	735	90°	7.281669°	9981	0.00	735.00	10000	0.000000	0.126747	-0.991935
laser 5	735	220°	7.281669°	9981	-563.04	-472.45	10000	-0.097094	-0.081472	-0.991935
laser 6	735	320°	7.281669°	9981	563.04	-472.45	10000	0.097094	-0.081472	-0.991935

Table 1. In terms of the values in the table the laser's positions are defined as $(x, y, z) = (r\cos\phi, r\sin\phi, F)$ and the laser's direction as: $(n_x, n_y, n_z) = (\sin\alpha\cos\phi, \sin\alpha\sin\phi, -\cos\alpha)$. D is the length of the laser beams between the lasers and the mirrors. The CCDs will be positioned perpendicular of the laser beams.

The red lines in Figure 1.left show the six laser beams going from the focal plane to the primary mirror. Figure 1.right also shows the same laser beams but from a top view. In this top view the x-axis points to the left and the y-axis points down. We can see that three lasers point to the outside edges of the primary mirror while the other three reflect back from the primary mirror itself.

- Option 2.

  In this case the CCDs that measure lasers 4 to 6 in Option 1 are moved to where the flat mirrors are. This shortens the air path for the lasers but has the disadvantage that in order to measure the primary mirror motion one would need a set of lasers pointing back to the focal plane.

- Option 3.

  In this option lasers 4 to 6 are replaced by BCAMs. Another set of three BCAMs are installed where the flat mirrors are in Option 1. These BCAMs are rigidly attached to the primary mirror and point back to the BCAMs attached to the camera near the focal plane.

- Option 4.

  In this option all lasers are replaced by BCAMs. The first three BCAM are attached to the camera at focal plane hight and the other three are attached close to the beginning of the camera. The coordinates for this set of BCAMs are given in Table 2. A companion set of six BCAMs are rigidly attached to the outside edge of the primary mirror. The BCAMs in this set points back to the BCAMs attached to the camera.

Table 2. Position and direction of BCAMs in Option 4. The other pairs of BCAMs are attached to the outside of the primary mirror and point back to the ones listed on the table.

	r	ϕ	α	D	x	y	z	n_x	n_y	n_z
BCAM 1	735	40°	7.281669°	9973	563.04	472.45	10000	0.097094	0.081472	-0.991935
BCAM 2	735	140°	7.281669°	9981	-563.04	472.45	10000	-0.097094	0.081472	-0.991935
BCAM 3	735	270°	7.281669°	9981	0.00	-735.00	10000	0.000000	-0.126747	-0.991935
BCAM 4	735	90°	8.089541°	8989	0.00	735.00	9000	0.000000	0.140721	-0.990049
BCAM 5	735	220°	8.089541°	8989	-563.04	-472.45	9000	-0.107798	-0.090453	-0.990049
BCAM 6	735	320°	8.089541°	8989	563.04	-472.45	9000	0.107798	-0.090453	-0.990049

The relation between the camera translations and rotations and the motion of the laser beams at the CCDs has been worked out for Options 1 to 4 in Appendix A. For small motions (on the order of a millimeter) the relation between the camera motion $\overrightarrow{X} = (\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z)^T$ and the CCD measurements $\overrightarrow{S} = (\Delta S_{x1}, ..., \Delta S_{x6}, \Delta S_{y1}, ..., \Delta S_{y6})^T$ is linear. Therefore, as in Appendix B, we can write

$$S_i = \sum_{\beta=1}^{6} C_{i\beta}\, X_\beta \tag{10}$$

The matrix C is given by Eqs 27, 31, 34 and 40 for Options 1 to 4 respectively. Since we have 12 CCD measurements and 6 camera degrees of freedom we can fit the CCD measurements. The details of the fit are given in Appendix B, the final result is a relation like

$$X_\alpha = \sum_{i=1}^{12} A_{\alpha i}\, S_i \tag{11}$$

For example for Option 1 the matrix A is given by

$$A^T = \begin{pmatrix} 0.215 & -0.259 & 0.337 & 0.000 & 0.000 & 0.000 \\ 0.215 & 0.259 & -0.337 & 0.000 & 0.000 & 0.000 \\ -0.334 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ -0.184 & 0.000 & 0.000 & 0.000 & 0.004 & 0.030 \\ 0.092 & -0.127 & 0.000 & -0.003 & -0.002 & 0.026 \\ 0.092 & 0.127 & 0.000 & 0.003 & -0.002 & 0.026 \\ -0.255 & -0.184 & -4.332 & 0.000 & 0.000 & 0.000 \\ 0.255 & -0.184 & -4.332 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.369 & -5.052 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.167 & 0.000 & 0.003 & 0.000 & 0.000 \\ -0.130 & -0.107 & 0.000 & -0.002 & 0.003 & 0.002 \\ 0.130 & -0.107 & 0.000 & -0.002 & -0.003 & -0.002 \end{pmatrix} \tag{12}$$

The CCD measurements S_i are all independent, therefore we can calculate the rms error of the camera motion by adding all errors in quadratures. That is

$$\sigma^2_{X\alpha} = \sum_{i=1}^{12} A^2_{\alpha i}\sigma^2_{Si} \tag{13}$$

where $\sigma_{X\alpha}$ are the rms errors of the camera motion and σ_{Si} are the rms errors of the CCD measurements. We will further assume that the CCD errors in x and y are the same. Also since the laser/CCD pairs split in two groups we will assume that we have only two errors, σ_1 for lasers 1 to 3 and σ_2 for lasers 4 to 6. Therefore we will split Eq. 13 in two pieces

$$\sigma^2_{X\alpha} = B^2_{\alpha 1}\,\sigma^2_1 + B^2_{\alpha 2}\,\sigma^2_2 \tag{14}$$

Table 3. Coefficients $B_{\alpha 1}$ and $B_{\alpha 2}$ as shown in Eq. 14.

	σ_x	σ_y	σ_z	$\sigma_{\theta x}$	$\sigma_{\theta y}$	$\sigma_{\theta x}$
Option 1	0.58 σ_1 0.29 σ_2	0.58 σ_1 0.29 σ_2	8.0 σ_1 0.0 σ_2	0.000 σ_1 0.006 σ_2	0.000 σ_1 0.006 σ_2	0.000 σ_1 0.047 σ_2
Option 2	0.29 σ_1 0.29 σ_2	0.29 σ_1 0.29 σ_2	2.0 σ_1 3.4 σ_2	0.006 σ_1 0.006 σ_2	0.006 σ_1 0.006 σ_2	0.000 σ_1 0.060 σ_2
Option 3	0.00 σ_1 0.58 Dσ_2/f	0.02 σ_1 0.58 Dσ_2/f	2.0 σ_1 3.4 Dσ_2/f	0.012 σ_1 0.012 Dσ_2/f	0.012 σ_1 0.012 Dσ_2/f	0.000 σ_1 0.163 Dσ_2/f
Option 4	0.60 Dσ_1/f 0.05 Dσ_2/f	0.61 Dσ_1/f 0.05 Dσ_2/f	2.1 Dσ_1/f 2.3 Dσ_2/f	0.119 Dσ_1/f 0.119 Dσ_2/f	0.119 Dσ_1/f 0.119 Dσ_2/f	0.081 Dσ_1/f 0.081 Dσ_2/f

The coefficient $B_{\alpha 1}$ and $B_{\alpha 2}$ are listed in Table 3 for the four options studied in this note. The values of D are given in Tables 1 and 2. For f we will use the value used in the "long-range" BCAMs[7] of f=0.15 meters.

Now to calculate the errors in the motion of the DECam camera we need to make some assumptions about the errors in the position measurement of the laser beams. The resolution of the "long-range" BCAMs is limited by air dispersion.[7] For measurement of up to 16 meters the beam spot at the CCD can be measured with an error of $\sigma = 0.5$ μm which translates into an angular resolution of σ/f=3.3 μrad. Since D is typically about 10 meters we will assume a BCAM resolution of about $D\sigma/f \approx 30$ μm.

We don't know yet what the effect of air dispersion will be in the other cases, but as a first approximation we will assume that it is the same as for BCAMs. Therefore we will assume a 30 μm resolution in all cases. When doing this the errors in the camera motion can be easily calculated. The results are shown in Table 4.

Table 4. DECam positioning errors (rms) assuming a laser beam position measurement error of 30 μm rms. The numbers in parenthesis give the ratio of the camera positioning error to the current hexapod specifications.

	σ_x	σ_y	σ_z	$\sigma_{\theta x}$	$\sigma_{\theta y}$	$\sigma_{\theta x}$
Option 1	19 μm (1.4)	19 μm (1.4)	240 μm (80)	0.2 arcsec (0.3)	0.2 arcsec (0.3)	1.4 arcsec
Option 2	12 μm (0.9)	12 μm (0.9)	118 μm (39)	0.3 arcsec (0.4)	0.3 arcsec (0.4)	1.8 arcsec
Option 3	17 μm (1.2)	17 μm (1.2)	118 μm (39)	0.5 arcsec (0.8)	0.5 arcsec (0.8)	4.9 arcsec
Option 4	18 μm (1.3)	18 μm (1.3)	93 μm (31)	5.0 arcsec (8.4)	5.0 arcsec (8.4)	3.4 arcsec

The numbers in parenthesis are the ratio of the resolutions listed on the table and the expected camera motion accuracy currently specified for the hexapod system (rms values of 14 μm, 3 μm and 0.6 arcsec for perpendicular translations, focusing and tip/tilt respectively). Three things are clear from the table:

1. The translation specification are essentially achieved.
2. The angular specifications are achieved in the first three options but not in the last one. This is not surprising given the small separation (1 meter) between the two sets of BCAMs.
3. The focusing errors are far from the hexapod requirements. This again is not surprising given that the lasers point almost vertically, and of course in the case that the lasers would point exactly vertical there would be no z resolution at all.

The effect of air dispersion can be reduced by averaging over time. In the case of BCAMs the resolution improves as[7] $n^{-0.2}$ in the vertical direction and $n^{-0.3}$ in the horizontal direction. Since in these measurements the closest surface to the camera's air path was a wall 30 cm to one side we will use the vertical scaling. The BCAM readings were taken every 1.5 seconds. If we could average over 100 seconds (assuming we use a 1.5 μm source and have it on all the time) we would gain a factor of $(100/1.5)^{-0.2}$=1/2.3. Using a faster device and averaging over more reading could potentially reduce the error even further.

4. REMAINING ISSUES

There are several issues that need to be understood in order to determine the best arrangement to measure the position of the camera with the required accuracy. As mentioned before the BCAM CCDs provide a centroid with ≈ 0.5 μm accuracy, so the CCD resolution is not going to be a problem. Dispersion in air will most likely determine the ultimate resolution of the system. The issues to be solved in the order in which they need to be understood are:

- Can we use infrared lasers and have the system ON all the time.
- How big is the air dispersion? How much can we gain by averaging?
- If the air dispersion is manageable, then we will need to optimize the laser beam shapes and integration times for optimal resolution.
- How best to provide the optimal laser beam shapes.
- Are the laser beam shapes stable enough over time, or will we need to measure them with a split mirror.
- Finally if the answer to the above question is positive we will need to find the best way to mount the lasers and CCDs to the camera.

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APPENDIX A. CAMERA POSITION MEASUREMENTS: DETAIL CALCULATIONS.

In this section we will study several arrangements of lasers and CCDs in order to try to determine the position of the DECam camera relative to the primary mirror. We will refer to the laser beam position measuring device as a CCD even though it can be any detector suitable for the measurement.

The lasers will be positioned at the primary mirror focal plane $z = F$, at a radius r and an angle ϕ, giving a laser position $\overrightarrow{x} = (r\cos\phi, r\sin\phi, F)$. The lasers will be pointing down towards the primary mirror at an angle α relative to the vertical axis. The unit vectors pointing in the direction of the laser beam are given by $\hat{n}_L = (\sin\alpha\cos\phi, \sin\alpha\sin\phi, -\cos\alpha)$.

A camera translation along the focal plane will move the lasers by an amount $\overrightarrow{\Delta}_T = (\Delta x, \Delta y, \Delta z)$. Rotations will also move the position of the lasers. For small angles rotations are given by

$$R = I + (M_x\,\Delta\theta_x + M_y\,\Delta\theta_y + M_z\,\Delta\theta_z) \tag{15}$$

where I is the identity matrix and

$$M_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\ ,\quad M_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}\ ,\text{ and }\ M_z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{16}$$

Since we are interested about the position of the focal plane we will perform translations and rotation around the center of the focal plane. That means that the laser position vectors we want to rotate are $\overrightarrow{x} = (r\cos\phi, r\sin\phi, 0)$. Therefore the change in the position of the lasers will be given by

$$\begin{aligned} \Delta(R\,\overrightarrow{x}) &= (M_x\,\Delta\theta_x + M_y\,\Delta\theta_y + M_z\,\Delta\theta_z)\,\overrightarrow{x} && (17) \\ &= (-r\sin\phi\,\Delta\theta_z, r\cos\phi\,\Delta\theta_z, r\sin\phi\,\Delta\theta_x - r\cos\phi\,\Delta\theta_y) && (18) \end{aligned}$$

Adding the translations $(\Delta x, \Delta y, \Delta z)$ we will have a total motion of the lasers given by

$$\overrightarrow{\Delta}_T = (\Delta x - r\sin\phi\,\Delta\theta_z\ ,\ \Delta y + r\cos\phi\,\Delta\theta_z\ ,\ \Delta z + r\sin\phi\,\Delta\theta_x - r\cos\phi\,\Delta\theta_y) \tag{19}$$

The change in the direction of the laser beams will be

$$\Delta(R\,\hat{n}_L) = (M_x\,\Delta\theta_x + M_y\,\Delta\theta_y + M_z\,\Delta\theta_z)\,\hat{n}_L = \begin{pmatrix} -c_\alpha\,\Delta\theta_y - s_\alpha\,s_\phi\,\Delta\theta_z \\ c_\alpha\,\Delta\theta_x + s_\alpha\,c_\phi\,\Delta\theta_z \\ s_\alpha\,s_\phi\,\Delta\theta_x - s_\alpha\,c_\phi\,\Delta\theta_y \end{pmatrix} \tag{20}$$

The CCDs will be positioned perpendicular to the laser beam. We will define the coordinate axis attached to the CCDs as follows: 1) the CCD will be in the x-y plane, 2) if we look at the CCD as the laser beam does then the x-axis points right and the y-axis points up. The z-axis will be coming out of the CCD. We will have CCDs at the focal plane and at the edge of the primary mirror. For the CCDs at the focal plane we will assume for simplicity that the laser and the CCD are at the same position. In reality the CCDs will have to be displaced relative to the laser, but this displacement is small in comparison to the distance between the focal plane and the primary mirror and can be ignored for now.

For the focal plane CCDs, the unit vectors of the CCD coordinate system are:

$$\begin{aligned} \hat{i}_F &= (-\sin\phi, \cos\phi, 0) \\ \hat{j}_F &= (\cos\alpha\cos\phi, \cos\alpha\sin\phi, \sin\alpha) \\ \hat{k}_F &= (\sin\alpha\cos\phi, \sin\alpha\sin\phi, -\cos\alpha) \end{aligned} \tag{21}$$

For the CCDs at the edge of the primary mirror, the unit vectors of the CCD coordinate system are:

$$
\begin{aligned}
\hat{i}_M &= (\sin\phi, -\cos\phi, 0) \\
\hat{j}_M &= (\cos\alpha\cos\phi, \cos\alpha\sin\phi, \sin\alpha) \\
\hat{k}_M &= (-\sin\alpha\cos\phi, -\sin\alpha\sin\phi, \cos\alpha)
\end{aligned}
\tag{22}
$$

A.1 Option 1

In this case lasers 1 to 3 will point to the primary mirror. Lasers 4 to 6 will points towards the outside edge of the mirror and the beam will be reflected from small flat mirrors solidly attached to the primary mirror. In both cases the reflected beams will be measured by a CCD right next to the laser. The laser coordinates are given in Table 1.

For lasers 1 to 3 we can calculate how the laser beams change at the CCDs when the camera moves as follows. Due to the focusing properties of the mirror, for a parallel displacement of the lasers the beam will come back to the same focal point. Since the CCDs are moving with the lasers the beam at the CCDs will be displaced by an amount $-\overrightarrow{\Delta}_T$, where $\overrightarrow{\Delta}_T$ is given by Eq. 19. For small angles and small displacements the primary mirror behaves as a lens of focal length F. Therefore if the lasers change by an angle $\Delta(R\,\hat{n}_L)$ (see Eq. 20) then the beam position at the CCD will move by $\overrightarrow{\Delta}_R = D\,\Delta(R\,\hat{n}_L)$, where $D \approx F$ is the distance between the point in the primary mirror where the laser beam hits and the CCD. Due to the rotational invariance of the primary mirror we have to set $\Delta\theta_z$ to zero in Eqs 19 and 20. Therefore the total beam motion at the CCD is given by $(-\overrightarrow{\Delta}_T + \overrightarrow{\Delta}_R)_{\Delta\theta_z=0}$. Now we use Eqs 21 to project onto the CCD coordinate system

$$
\begin{aligned}
\Delta S_x^{(1-3)} &= (-\overrightarrow{\Delta}_T + \overrightarrow{\Delta}_R)_{\Delta\theta_z=0} \cdot \hat{i}_F \\
&= s_\phi\,\Delta x - c_\phi\,\Delta y + D\,c_\alpha\,c_\phi\,\Delta\theta_x + D\,c_\alpha\,s_\phi\,\Delta\theta_y
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
\Delta S_y^{(1-3)} &= (-\overrightarrow{\Delta}_T + \overrightarrow{\Delta}_R)_{\Delta\theta_z=0} \cdot \hat{j}_F \\
&= -c_\alpha\,c_\phi\,\Delta x - c_\alpha\,s_\phi\,\Delta y - s_\alpha\,\Delta z + (-r\,s_\alpha\,s_\phi + D\,s_\phi)\,\Delta\theta_x + (r\,s_\alpha\,c_\phi - D\,c_\phi)\,\Delta\theta_y
\end{aligned}
\tag{24}
$$

For lasers 4 to 6 the beam reflects from a flat mirror. Since the laser and CCD move together when the camera is translated along x, y or z the reflected laser beam and the CCD will move the same way giving $\Delta S_x = \Delta S_y = 0$. For rotations we will have $\overrightarrow{\Delta}_R = 2D\,\Delta(R\,\hat{n}_L)$ because we are now reflecting from a flat mirror. Also since the flat mirror breaks the rotational symmetry of the primary mirror the beam at the CCD will move as the camera is rotated around the z-axis. The projections onto the CCD coordinate system will be

$$
\Delta S_x^{(4-6)} = \overrightarrow{\Delta}_R \cdot \hat{i}_F = 2D\,c_\alpha\,c_\phi\,\Delta\theta_x + 2D\,c_\alpha\,s_\phi\,\Delta\theta_y + 2D\,s_\alpha\Delta\theta_z \tag{25}
$$

$$
\Delta S_y^{(4-6)} = \overrightarrow{\Delta}_R \cdot \hat{j}_F = 2D\,s_\phi\,\Delta\theta_x - 2D\,c_\phi\,\Delta\theta_y \tag{26}
$$

We will assume now that the primary mirror is a parabola with focal length F=10 meters. In units of micrometers and arcseconds and the laser positions given in Table 1 we get

$$
\begin{pmatrix}
\Delta S_{x_1} \\ \Delta S_{x_2} \\ \Delta S_{x_3} \\ \Delta S_{x_4} \\ \Delta S_{x_5} \\ \Delta S_{x_6} \\ \hline
\Delta S_{y_1} \\ \Delta S_{y_2} \\ \Delta S_{y_3} \\ \Delta S_{y_4} \\ \Delta S_{y_5} \\ \Delta S_{y_6}
\end{pmatrix}
=
\left(\begin{array}{rrrrrr}
0.643 & -0.766 & 0.000 & 36.940 & 30.996 & 0.000 \\
0.643 & 0.766 & 0.000 & -36.940 & 30.996 & 0.000 \\
-1.000 & 0.000 & 0.000 & 0.000 & -48.222 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 95.998 & 12.266 \\
0.000 & 0.000 & 0.000 & -73.539 & -61.706 & 12.266 \\
0.000 & 0.000 & 0.000 & 73.539 & -61.706 & 12.266 \\ \hline
-0.764 & -0.641 & -0.073 & 30.912 & -36.840 & 0.000 \\
0.764 & -0.641 & -0.073 & 30.912 & 36.840 & 0.000 \\
0.000 & 0.997 & -0.073 & -48.091 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 96.779 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & -62.208 & 74.137 & 0.000 \\
0.000 & 0.000 & 0.000 & -62.208 & -74.137 & 0.000
\end{array}\right)
\begin{pmatrix}
\Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z
\end{pmatrix}
\tag{27}
$$

We have also calculated the same matrix by ray tracing, the results is

$$
\left(\begin{array}{c}
\Delta S_{x_1} \\ \Delta S_{x_2} \\ \Delta S_{x_3} \\ \Delta S_{x_4} \\ \Delta S_{x_5} \\ \Delta S_{x_6} \\ \hline
\Delta S_{y_1} \\ \Delta S_{y_2} \\ \Delta S_{y_3} \\ \Delta S_{y_4} \\ \Delta S_{y_5} \\ \Delta S_{y_6}
\end{array}\right)
=
\left(\begin{array}{rrrrrr}
0.639 & -0.762 & 0.000 & 37.137 & 31.161 & 0.000 \\
0.639 & 0.762 & 0.000 & -37.137 & 31.161 & 0.000 \\
-0.995 & 0.000 & 0.000 & 0.000 & -48.479 & 0.000 \\
0.000 & 0.000 & 0.000 & 0.000 & 95.990 & 12.265 \\
0.000 & 0.000 & 0.000 & -73.533 & -61.702 & 12.265 \\
0.000 & 0.000 & 0.000 & 73.533 & -61.701 & 12.265 \\ \hline
-0.756 & -0.634 & -0.072 & 31.244 & -37.235 & 0.000 \\
0.756 & -0.634 & -0.072 & 31.244 & 37.235 & 0.000 \\
0.000 & 0.987 & -0.072 & -48.606 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & 96.771 & 0.000 & 0.000 \\
0.000 & 0.000 & 0.000 & -62.203 & 74.131 & 0.000 \\
0.000 & 0.000 & 0.000 & -62.203 & -74.131 & 0.000
\end{array}\right)
\left(\begin{array}{c}
\Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z
\end{array}\right)
\tag{28}
$$

And as expected the differences are not very big (≤ 1.1 %).

A.2 Option 2

In this case lasers 1 to 3 will point to the primary mirror as in Option 1. Lasers 4 to 6 will points towards the outside edge of the mirror but instead of being reflected the laser beam will be measured by a CCD placed where the flat mirrors were located in Option 1.

The relation between the camera motion and the CCD measurements for lasers 1 to 3 will be the same an in Eqs 23 and 24. For lasers 4 to 6 there will be two changes: 1) for rotations we have to multiply by D instead of $2D$, and 2) since the CCDs don't move with the laser we will see the camera translations $\overrightarrow{\Delta}_T$. Now we have to project onto the CCD coordinate system given by Eq. 22.

$$
\begin{aligned}
\Delta S_x^{(4-6)} &= (\overrightarrow{\Delta}_T + \overrightarrow{\Delta}_R) \cdot \hat{\imath}_M \\
&= s_\phi\,\Delta x - c_\phi\,\Delta y - D\,c_\alpha\,c_\phi\,\Delta\theta_x - D\,c_\alpha\,s_\phi\,\Delta\theta_y - (r + D\,s_\alpha)\,\Delta\theta_z && (29) \\
\Delta S_y^{(4-6)} &= (\overrightarrow{\Delta}_T + \overrightarrow{\Delta}_R) \cdot \hat{\jmath}_M \\
&= c_\alpha\,c_\phi\,\Delta x + c_\alpha\,s_\phi\,\Delta y + s_\alpha\,\Delta z + (r\,s_\alpha\,s_\phi + D\,s_\phi)\,\Delta\theta_x - (r\,s_\alpha\,c_\phi + D\,c_\phi)\,\Delta\theta_y && (30)
\end{aligned}
$$

This option is attractive because the laser path is shorter which means that the dispersion in air will have a smaller effect. On the other hand if the mirror can rotate relative to the camera then we will have to add another laser pointing towards the focal plane with the corresponding CCD. The reason is that if the camera is fixed a rotation of the mirror will only rotate the CCD attached to it very slightly and we will see no effect.

If as the primary mirror we use a parabola with focal length F=10 meters then in units of micrometers and arcseconds and the laser positions given in Table 1 we get

$$
\left(\begin{array}{c}
\Delta S_{x_1} \\ \Delta S_{x_2} \\ \Delta S_{x_3} \\ \Delta S_{x_4} \\ \Delta S_{x_5} \\ \Delta S_{x_6} \\ \hline
\Delta S_{y_1} \\ \Delta S_{y_2} \\ \Delta S_{y_3} \\ \Delta S_{y_4} \\ \Delta S_{y_5} \\ \Delta S_{y_6}
\end{array}\right)
=
\left(\begin{array}{rrrrrr}
0.643 & -0.766 & 0.000 & 36.940 & 30.996 & 0.000 \\
0.643 & 0.766 & 0.000 & -36.940 & 30.996 & 0.000 \\
-1.000 & 0.000 & 0.000 & 0.000 & -48.222 & 0.000 \\
1.000 & 0.000 & 0.000 & 0.000 & -47.999 & -9.697 \\
-0.643 & 0.766 & 0.000 & 36.769 & 30.853 & -9.697 \\
-0.643 & -0.766 & 0.000 & -36.769 & 30.853 & -9.697 \\ \hline
-0.764 & -0.641 & -0.073 & 30.912 & -36.840 & 0.000 \\
0.764 & -0.641 & -0.073 & 30.912 & 36.840 & 0.000 \\
0.000 & 0.997 & -0.073 & -48.091 & 0.000 & 0.000 \\
0.000 & 0.992 & 0.127 & 48.841 & 0.000 & 0.000 \\
-0.760 & -0.638 & 0.127 & -31.394 & 37.414 & 0.000 \\
0.760 & -0.638 & 0.127 & -31.394 & -37.414 & 0.000
\end{array}\right)
\left(\begin{array}{c}
\Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z
\end{array}\right)
\tag{31}
$$

A comparison with a ray tracing program gives differences that are smaller than 1.1 %.

A.3 Option 3

In this case lasers 1 to 3 will point to the primary mirror as in Options 1 and 2. Lasers 4 to 6 will be replaced by BCAM cameras. If one adds a lens of focal length f at a distance f in front of the CCD then one measures the beam angle instead of the beam position. This is what the "Brandeis CCD Angle Monitor[7,8]" (or BCAM) does. Each BCAM has two 875 nm LEDs, a light diffuser, a lens and a CCD. When pointed at each other they can measure their relative transverse position by looking at the beam spot displacement on the CCD. Since the beam going through the center of a lens doesn't deflect, the distance D between two BCAMs facing each other, their displacement $\overrightarrow{\Delta}_T$ and the beam displacement $\overrightarrow{\Delta}_S$ at the CCD are related by $\overrightarrow{\Delta}_T/D = -\overrightarrow{\Delta}_S/f$. The BCAM light beams have a very large divergence and the part of the beam that is seen by the CCD is determined by a small aperture in front of the lens which means that the position measurements are insensitive to changes in the BCAM angles.

The idea will be to place one BCAM at the focal plane and another where the flat mirrors where in Option 1. The camera translations will move the beam at the CCD by $-f/D\,\overrightarrow{\Delta}_T$, with $\overrightarrow{\Delta}_T$ given by Eq. 19. Projecting into the CCD coordinate system given in Eq. 22

$$\begin{aligned}
\Delta S_x^{(4-6)} &= (\overrightarrow{\Delta}_T + \overrightarrow{\Delta}_R)\cdot \hat{\imath}_M = -f/D\,(\,s_\phi\,\Delta x - \,c_\phi\,\Delta y - r\,\Delta\theta_z) && (32)\\
\Delta S_y^{(4-6)} &= (\overrightarrow{\Delta}_T + \overrightarrow{\Delta}_R)\cdot \hat{\jmath}_M = -f/D\,(c_\alpha\,c_\phi\,\Delta x + c_\alpha\,s_\phi\,\Delta y + s_\alpha\,\Delta z + r\,s_\alpha\,s_\phi\,\Delta\theta_x - r s_\alpha\,c_\phi\,\Delta\theta_y) && (33)
\end{aligned}$$

As before we use a parabola with focal length F=10 meters for the primary mirror, and a focal length f=0.150 meters for BCAMs. If we use units of micrometers and arcseconds and the laser positions given in Table 1 we get

$$\begin{pmatrix} \Delta S_{x_1} \\ \Delta S_{x_2} \\ \Delta S_{x_3} \\ (D/f)\,\Delta S_{x_4} \\ (D/f)\,\Delta S_{x_5} \\ (D/f)\,\Delta S_{x_6} \\ \hline \Delta S_{y_1} \\ \Delta S_{y_2} \\ \Delta S_{y_3} \\ (D/f)\,\Delta S_{y_4} \\ (D/f)\,\Delta S_{y_5} \\ (D/f)\,\Delta S_{y_6} \end{pmatrix} = \left(\begin{array}{rrrrrr} 0.643 & -0.766 & 0.000 & 36.940 & 30.996 & 0.000 \\ 0.643 & 0.766 & 0.000 & -36.940 & 30.996 & 0.000 \\ -1.000 & 0.000 & 0.000 & 0.000 & -48.222 & 0.000 \\ -1.000 & 0.000 & 0.000 & 0.000 & 0.000 & 3.563 \\ 0.643 & -0.766 & 0.000 & 0.000 & 0.000 & 3.563 \\ 0.643 & 0.766 & 0.000 & 0.000 & 0.000 & 3.563 \\ \hline -0.764 & -0.641 & -0.073 & 30.912 & -36.840 & 0.000 \\ 0.764 & -0.641 & -0.073 & 30.912 & 36.840 & 0.000 \\ 0.000 & 0.997 & -0.073 & -48.091 & 0.000 & 0.000 \\ 0.000 & -0.992 & -0.127 & -0.452 & 0.000 & 0.000 \\ 0.760 & 0.638 & -0.127 & 0.290 & -0.346 & 0.000 \\ -0.760 & 0.638 & -0.127 & 0.290 & 0.346 & 0.000 \end{array}\right) \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \qquad (34)$$

A comparison with a ray tracing program gives differences that are smaller than 1.1 %.

A.4 Option 4

In this case we will investigate the possibility of using all BCAMs. Lasers 1 to 3 will be in the same configuration as lasers 4 to 6 in Option 3. That is

$$\begin{aligned}
\Delta S_x^{(1-3)} &= -f/D\,(\,s_\phi\,\Delta x - \,c_\phi\,\Delta y - r\,\Delta\theta_z) && (35)\\
\Delta S_y^{(1-3)} &= -f/D\,(c_\alpha\,c_\phi\,\Delta x + c_\alpha\,s_\phi\,\Delta y + s_\alpha\,\Delta z + r\,s_\alpha\,s_\phi\,\Delta\theta_x - r s_\alpha\,c_\phi\,\Delta\theta_y) && (36)
\end{aligned}$$

The other group of lasers have to be in a different configuration to break the degeneracy. The only other configuration we know of that will do that is to put them at a different z. So we will assume that they are at a distance L below the focal plane. Then the BCAM's positions will go from $\overrightarrow{x} = (r\cos\phi, r\sin\phi, 0)$ to $\overrightarrow{x} = (r\cos\phi, r\sin\phi, L)$. Using Eq. 17 we can easily see that there is an extra translation given by $\overrightarrow{\Delta}_T^{(L)} = (-L\,\Delta\theta_x, L\,\Delta\theta_y, 0)$. Projecting into the CCD coordinate system we get

$$\begin{aligned}
\Delta S_x^{(4-6)} &= \Delta S_x^{(1-3)} - f/D\,\overrightarrow{\Delta}_T^{(L)}\cdot\hat{\imath}_M = \Delta S_x^{(1-3)} - f/D\,(L\,c_\phi\,\Delta\theta_x + L\,s_\phi\,\Delta\theta_y) && (37)\\
\Delta S_y^{(4-6)} &= \Delta S_y^{(1-3)} - f/D\,\overrightarrow{\Delta}_T^{(L)}\cdot\hat{\jmath}_M = \Delta S_y^{(1-3)} - f/D\,(-L\,c_\alpha\,s_\phi\,\Delta\theta_x + L\,c_\alpha\,c_\phi\,\Delta\theta_y) && (38)\\
& && (39)
\end{aligned}$$

If we use units of micrometers and arcseconds and $L = -1$ meters we get

$$\begin{pmatrix} (D/f)\,\Delta S_{x_1} \\ (D/f)\,\Delta S_{x_2} \\ (D/f)\,\Delta S_{x_3} \\ (D/f)\,\Delta S_{x_4} \\ (D/f)\,\Delta S_{x_5} \\ (D/f)\,\Delta S_{x_6} \\ \hline (D/f)\,\Delta S_{y_1} \\ (D/f)\,\Delta S_{y_2} \\ (D/f)\,\Delta S_{y_3} \\ (D/f)\,\Delta S_{y_4} \\ (D/f)\,\Delta S_{y_5} \\ (D/f)\,\Delta S_{y_6} \end{pmatrix} = \left(\begin{array}{rrrrrr} -0.643 & 0.766 & 0.000 & 0.000 & 0.000 & 3.563 \\ -0.643 & -0.766 & 0.000 & 0.000 & 0.000 & 3.563 \\ 1.000 & 0.000 & 0.000 & 0.000 & 0.000 & 3.563 \\ -1.000 & 0.000 & 0.000 & 0.000 & 4.848 & 3.563 \\ 0.643 & -0.766 & 0.000 & -3.714 & -3.116 & 3.563 \\ 0.643 & 0.766 & 0.000 & 3.714 & -3.116 & 3.563 \\ \hline -0.760 & -0.638 & -0.127 & -0.290 & 0.346 & 0.000 \\ 0.760 & -0.638 & -0.127 & -0.290 & -0.346 & 0.000 \\ 0.000 & 0.992 & -0.127 & 0.452 & 0.000 & 0.000 \\ 0.000 & -0.990 & -0.141 & -5.301 & 0.000 & 0.000 \\ 0.758 & 0.636 & -0.141 & 3.408 & -4.061 & 0.000 \\ -0.758 & 0.636 & -0.141 & 3.408 & 4.061 & 0.000 \end{array}\right) \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \tag{40}$$

APPENDIX B. LINEAR FIT

In this section we will write down the formulas we used in the linear fit of the CCD measurements. In Appendix A we calculated the relation between the camera motion $\overrightarrow{X} = (\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z)^T$ and the CCD measurements $\overrightarrow{S} = (\Delta S_{x1}, ..., \Delta S_{xn}, \Delta S_{y1}, ..., \Delta S_{yn})^T$. The relation between the two is given by a matrix C:

$$S_i^{predic} = \sum_{\beta=1}^{6} C_{i\beta}\, X_\beta \tag{41}$$

We determine the position of the camera by fitting to the CCD measurements $\overrightarrow{S}$

$$\chi^2 = \sum_{i=1}^{2n} \frac{1}{\sigma_i^2} \left(S_i - \sum_{\beta=1}^{6} C_{i\beta}\, X_\beta \right)^2 \tag{42}$$

Minimizing χ^2 we get

$$\frac{\partial \chi^2}{\partial X_\alpha} = -\sum_{i=1}^{2n} \frac{2C_{i\alpha}}{\sigma_i^2} \left(S_i - \sum_{\beta=1}^{6} C_{i\beta}\, X_\beta \right) = 0 \tag{43}$$

Or

$$\sum_{i=1}^{2n} \frac{C_{i\alpha} S_i}{\sigma_i^2} = \sum_{i=1}^{2n} \sum_{\beta=1}^{6} \frac{C_{i\alpha} C_{i\beta}}{\sigma_i^2} X_\beta = \sum_{\beta=1}^{6} \left(\sum_{i=1}^{2n} \frac{C_{i\alpha} C_{i\beta}}{\sigma_i^2} \right) X_\beta = \sum_{\beta=1}^{6} M_{\alpha\beta} X_\beta \tag{44}$$

where we have written

$$M_{\alpha\beta} = \sum_{i=1}^{2n} \frac{C_{i\alpha} C_{i\beta}}{\sigma_i^2} \tag{45}$$

Inverting $M_{\alpha\beta}$ and multiplying Eq. 44 by $M_{\alpha\beta}^{-1}$ we get

$$X_\alpha = \sum_{\beta=1}^{6} M_{\alpha\beta}^{-1} \sum_{i=1}^{2n} \frac{C_{i\beta} S_i}{\sigma_i^2} = \sum_{i=1}^{2n} \left(\sum_{\beta=1}^{6} \frac{M_{\alpha\beta}^{-1} C_{i\beta}}{\sigma_i^2} \right) S_i = \sum_{i=1}^{2n} A_{\alpha i}\, S_i \tag{46}$$

where we have written

$$A_{\alpha i} = \sum_{\beta=1}^{6} \frac{M_{\alpha\beta}^{-1} C_{i\beta}}{\sigma_i^2} \tag{47}$$

Now assuming that all the CCD measurement are independent we can calculate the rms error in the camera position by adding each contribution in quadratures. That is

$$\sigma_{x\alpha} = \sqrt{\sum_{i=1}^{2n} A_{\alpha i}^2 \sigma_{Si}^2} \tag{48}$$

REFERENCES

1. "The Dark Energy Camera (DECam) Project. Conceptual Design Report." Version 2.0. April 23, 2007.
2. "Focus and Alignment: Current Status." Mike Gladders, docdb September 17, 2007.
3. "The Focus and Alignment Sensors of the Dark Energy Camera." Mike Gadders, docdb September 17, 2007.
4. "Dark Energy Camera Specifications and Technical Requirements." Version 1.0. Tim Abbott et al, October 3, 2007.
5. "Request for Information (RFI). DECam Position Adjustment System." R. French Leger, Dave McGinnis and Andy Stefanik. July 12, 2007 - Revision 2.
6. Darren DePoy, private communication.
7. "The BCAM Camera." Kevan S. Hashemi and James R. Bensinger, ATLAS internal note, 15 September 2000.
8. "A novel solution for various monitoring applications at CERN." F. Lackner, P. H. Osanna, W. Riengler, H. Kopetz, Measurement Science Review, Volume 6, Section 1, No. 3, 2006.
9. "Precision Angle Monitors (BCAM) for Real-Time Telescope Alignment." David Gerdes, docdb May 22, 2007.